Improved GUT and SUSY breaking by the same field [1]

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Abstract

In a previous paper [1], we presented a model in which the *same* modulus field breaks SUSY and a simple GUT gauge group, and which has *dynamical* origins for *both* SUSY breaking and GUT scales. In this model, the supergravity (SUGRA) and gauge mediated contributions to MSSM scalar and gaugino masses are comparable – this enables a realistic spectrum to be attained since the gauge mediated contribution to the right-handed (RH) slepton $(\text{mass})^2$ (at the weak scale) by itself (i.e., neglecting SUGRA contribution to sfermion *and* gaugino masses) is negative. But, in general, the SUGRA contribution to sfermion masses (from non-renormalizable contact Kähler terms) leads to flavor violation. In this paper, we use the recently proposed idea of gaugino mediated SUSY breaking ($\tilde{g}$MSB) to improve the above model. With MSSM matter and SUSY breaking fields localized on separate branes in an extra dimension of size $R \sim 5 M_{Pl}^{-1}$ (in which gauge fields propagate), the SUGRA contribution to sfermion masses (which violates flavor) is suppressed. As in 4 dimensions, MSSM gauginos acquire *non*-universal masses from both SUGRA and gauge mediation – gaugino masses (in particular the SUGRA contribution to gaugino masses), in turn, generate acceptable sfermion masses through renormalization group evolution; the phenomenology is discussed briefly. We also point out that a) in models where SUSY is broken by a GUT non-singlet field, there is, in general, a contribution to MSSM gaugino (and scalar) masses from the coupling to heavy gauge multiplet which *might* be *comparable* to the SUGRA contribution and b) models of gauge mediation proposed earlier which also have negative RH slepton $(\text{mass})^2$ can be rendered viable using the $\tilde{g}$MSB idea.

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1   Introduction

Two central issues in supersymmetric grand unified theories (SUSY-GUT's) are a) mechanism of SUSY breaking (including the origin of the SUSY breaking scale) and mediation of SUSY breaking to the SM superpartners and b) mechanism of GUT symmetry breaking (down to the SM gauge group) and the origin of the GUT scale (denoted by M_{GUT}) $\sim 2 \times 10^{16}$ GeV (the energy scale at which SM gauge couplings unify with the MSSM particle content).

In [1], we presented a model in which both these symmetries (SUSY and a *simple* GUT gauge group) are broken by the same modulus field (i.e., by the *same* scalar potential). A non-zero vev for the F-component of this field is generated *dynamically* breaking SUSY. The GUT scale which is the vev of the scalar (A-)component of the *same* field (at the minimum of the potential) is determined (*dynamically*) by an "inverted hierarchy" mechanism. Therefore the GUT scale is *naturally* both larger than the SUSY breaking scale (which is required for the vev of the scalar component to be *calculable* in perturbation theory) *and* smaller than the Planck scale. This is the first example of its kind in the literature. [3]

In this model, there are two comparable contributions to the MSSM scalar and gaugino masses – one is mediated by supergravity (SUGRA) and the other by gauge interactions. This is crucial to achieving a realistic sfermion mass spectrum since if we neglect the SUGRA contribution, then the (gauge mediated contribution to) the right-handed (RH) slepton (mass)^2 (at the weak scale) is negative. However, the SUGRA contribution to sfermion (mass)^2 is, in general, arbitrary in flavor space giving unacceptable rates for flavor changing neutral currents (FCNC's). We will argue that, in general, this "problem" will be present in *any* model where SUSY and a GUT symmetry are broken by the same field.

In this paper, we show how the above model can be "improved", i.e., how "flavor-conserving" and positive sfermion (mass)^2 can be attained using the recently proposed idea of gaugino mediated SUSY breaking [4, 5].

2   Review of model and the problem

2.1   Model

We begin with a brief review of the model of [1]. The gauge group is:

$$SU(6)_{GUT} \times SU(6)_S \tag{1}$$

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[3]In [2, 3] also, SUSY and a GUT gauge group are broken by the same field. However, in [2], SUSY breaking is *not* dynamical so that a very small superpotential coupling is required to explain the smallness of the SUSY breaking scale compared to M_{Pl} and also if all superpotential couplings are of the same order, then $M_{GUT} \sim M_{Pl}$. In [3], the GUT gauge group is *not* simple so that gauge coupling unification is *not* a prediction of the model and also an assumption about a (*non-calculable*) Kähler potential is required for the model to work.

	$SU(6)_{GUT}$	$SU(6)_S$
Σ	**35**	**1**
$Q, \bar{Q}$	**6**, $\bar{\mathbf{6}}$	**6**, $\bar{\mathbf{6}}$
$H_{1,2}, \bar{H}_{1,2}$	**6**, $\bar{\mathbf{6}}$	**1**
$h_{1,2}, \bar{h}_{1,2}$	**6**, $\bar{\mathbf{6}}$	**1**
N_i	**15**	**1**
$\bar{P}_{1,2i}$	$\bar{\mathbf{5}}$	**1**
$Y, \bar{Y}$	**15**, $\bar{\mathbf{15}}$	**1**
$S_{1,2,3}, X_{1,2}, \bar{X}_{1,2}$	**1**	**1**

Table 1: The particle content of the model. $i = 1, 2, 3$ is a generational index.

and the field content is listed in Table 1.

The core part of the superpotential is:

$$W_1 = \lambda_Q \Sigma Q \bar{Q} + \frac{\lambda_\Sigma}{3} \Sigma^3. \tag{2}$$

Along the flat direction parametrized by tr Σ^2, the vev of Σ is:

$$\langle \Sigma \rangle = \frac{v}{\sqrt{12}} \, \text{diag}[1, 1, 1, -1, -1, -1] \tag{3}$$

which breaks $SU(6)_{GUT}$ to $SU(3) \times SU(3) \times U(1)$ at the scale v. We identify one unbroken $SU(3)$ with $SU(3)_{color}$ and show later how to break the other $SU(3) \times U(1)$ to $SU(2)_{weak} \times U(1)_Y$ at the same scale (v). Thus we identify the value of v at the minimum of the potential (see later) with the GUT scale (i.e., the scale at which SM gauge couplings meet). The Σ vev gives mass to $Q, \bar{Q}$ so that below the scale v, the only massless fields are the flat direction v (we will denote both the chiral superfield and its scalar component by v) and the pure gauge theory $SU(6)_S$ (the other components of Σ either get a mass through the Σ^3 term or are eaten by the broken generators of $SU(6)_{GUT}$; see [1] for details).

The pure $SU(6)_S$ gauge theory undergoes gaugino condensation and generates the superpotential:

$$W = 6\Lambda_L^3 = \sqrt{3}\lambda_Q \Lambda^2 v, \tag{4}$$

where Λ and Λ_L are the dynamical scales of the high and low energy $SU(6)_S$, respectively (they are related by one-loop matching at the scale v, assuming $v \gg \Lambda$). Thus, below the scale Λ_L, the only massless field is v with the above superpotential so that SUSY is broken since $F_v \sim \Lambda^2$. But, with the canonical (tree-level) Kähler potential ($v^\dagger v$), the vev of the scalar (A-)component of v (i.e., the GUT scale) is undetermined since the scalar potential is flat $\sim \Lambda^4$. The dominant

corrections to the Kähler potential (and hence the scalar potential) are given by the wavefunction renormalization of Σ (denoted by Z) so that:

$$V(v) \sim \frac{\Lambda^4}{Z(v)}. \tag{5}$$

Since $v \gg \Lambda$, we can compute $Z(v)$ using perturbation theory. In renormalization group (RG) evolution, at one-loop, $Z(v)$ receives contributions from the Yukawa coupling(s) $(\lambda_{\Sigma,Q})$ and the $SU(6)_{GUT}$ gauge coupling – the former (latter) tends to decrease (increase) $Z(v)$ as v increases. Thus, if the gauge coupling dominates at small v whereas the Yukawa coupling is larger at high scales (as is natural if $SU(6)_{GUT}$ is asymptotically free), then the potential can develop a minimum. Furthermore, the minimum can be (naturally) at a value of $v \gg \Lambda$ since Z and both couplings depend logarithmically on v: at the minimum, we require that $v \sim 10^{16}$ GeV (to obtain the correct GUT scale) whereas $\Lambda \sim 10^{10}-10^{11}$ GeV so that MSSM sparticles have masses ~ 100 GeV $- 1$ TeV (see later). [4] Thus, this inverted hierarchy mechanism [6] can generate a GUT scale, M_{GUT}, much larger than the SUSY breaking scale, Λ (which is also required for the perturbative calculation mentioned above to be valid) [5]. We can also view this as "generating" the GUT scale from the Planck scale ($M_{Pl} \sim 2 \times 10^{18}$ GeV) as follows. We can choose $Z = 1$ (canonical normalization) at M_{Pl} and RG evolve Z to lower energies. Since Z and the gauge and Yukawa couplings vary logarithmically ("slowly") with energy, (we can choose $O(1)$ couplings at M_{Pl} such that) Z reaches a maximum at an energy scale $\sim M_{GUT}$ which is "much" smaller (i.e., by two orders of magnitude) than M_{Pl} [1].

To break the other $SU(3) \times U(1)$ to $SU(2)_{weak} \times U(1)_Y$ (and to get the usual light Higgs doublets) we add:

$$W_2 = \sum_{i=1}^{2} S_i(H_i\bar{H}_i - \Sigma^2), \tag{6}$$

$$W_3 = \sum_{i=1}^{2} H_i(\Sigma + X_i)\bar{h}_i + \sum_{i=1}^{2} \bar{H}_i(\Sigma + \bar{X}_i)h_i, \tag{7}$$

$$W_4 = \frac{1}{M}\left(\left(H_1\bar{H}_1\right)\left(H_2\bar{H}_2\right) - \left(H_1\bar{H}_2\right)\left(H_2\bar{H}_1\right)\right), \tag{8}$$

$$W_5 = S_3\left(H_1\bar{H}_2 - H_2\bar{H}_1\right). \tag{9}$$

The role of these W's is as follows (for details, see [1]). W_2 forces H, $\bar{H}$ to acquire vev's. With $\langle H \rangle = \langle \bar{H} \rangle \sim v\,(1,0,0,0,0,0)$, $SU(3) \times U(1)$ is broken to $SU(2) \times U(1)$. W_3 forces X, $\bar{X}$ to acquire vev's such that only the triplets in H's get a mass with those in $\bar{h}$'s (this is the

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[4]This is only a local minimum since there is a supersymmetric minimum with $\langle \Sigma \rangle \sim \langle Q \rangle \sim \langle \bar{Q} \rangle \sim \Lambda$ [1]. However since, at the local minimum $v \gg \Lambda$ the tunneling rate from the local minimum to the "true" vacuum is very small.

[5]It should be possible to contruct models based on (say) $SO(10)$ along similar lines.

"sliding singlet" mechanism). [6] A pair of doublets in H, $\bar{H}$ is eaten in the gauge symmetry breaking while a pair of doublets in h, $\bar{h}$ gets a mass with doublets in Σ (due to the H, $\bar{H}$ vev). This leaves two pairs of doublets massless: the one in H, $\bar{H}$ acquires mass through W_4 and the one in h, $\bar{h}$ is the usual pair of Higgs doublets. W_5 gives a required constraint between the vev's of H_1, $\bar{H}_1$ and H_2, $\bar{H}_2$.

To complete the model, the SM quarks and leptons are obtained through:

$$
\begin{aligned}
W_6 \;=\;& N_i(\bar{P}_{1j}\bar{H}_1 + \bar{P}_{2j}\bar{H}_2) + N_i(\bar{P}_{1j}\bar{h}_1 + \bar{P}_{2j}\bar{h}_2) \\
&+ N_i N_j Y + (X_1 + X_2)Y\bar{Y} + \bar{Y}(H_1 h_1 - H_2 h_2).
\end{aligned}
\tag{10}
$$

For each generation, the vev's of $\bar{H}$'s gives a mass to one combination of $\bar{\mathbf{5}}$ (under $SU(5)$) in $\bar{P}_{1,2}$ with the $\mathbf{5}$ (under $SU(5)$) of N leaving one $\bar{\mathbf{5}}$ in $\bar{P}_{1,2}$ and $\mathbf{10}$ in N massless – these are the quarks and leptons. The other terms in W_6 generate their Yukawa couplings.

2.2   Problem with sfermion spectrum

The MSSM scalars and gauginos acquire masses $\sim [\alpha_{GUT}/\pi]\,[F_v/M_{GUT}] \sim 10^{-2}\,[F_v/M_{GUT}]$ through gauge mediation (GM) by coupling at one or two-loops to two sources [7]: one is the "matter" messengers – the Q, $\bar{Q}$ fields and the heavy components of H, $\bar{H}$, h and $\bar{h}$ fields which as usual have a SUSY breaking mass spectrum due to the coupling to v. [8] The other source, usually referred to as "gauge" messengers, is the heavy part of the $SU(6)_{GUT}$ gauge multiplet which also has a SUSY breaking spectrum since the field(s) breaking the GUT gauge group (Σ, H and $\bar{H}$) have a non-zero F-component.

Using the technique of [7], the MSSM gaugino masses are given by (A denotes the gauge group):

$$
M_A(\mu) \approx \frac{\alpha_A(\mu)}{4\pi}\frac{F_v}{v}\,(b_A - b_6)\,,
\tag{11}
$$

where b_A's are the beta functions of the SM gauge groups below the GUT scale and b_6 is the beta function of the $SU(6)_{GUT}$ above the GUT scale. The gaugino masses are *non*-universal since the messengers (both gauge and matter) are *not* in complete GUT multiplets.

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[6]Any symmetries which allow the terms Σ^3, $S\Sigma^2$ and $SH\bar{H}$ (which we need to obtain the desired vev's), also allow the term $\Sigma H\bar{H}$ which spoils the above pattern of vev's; in addition, there might be higher dimension operators (allowed by the same symmetries) which might spoil the sliding singlet mechanism and/or the pattern of vev's. Thus, this model is only "technically" natural, i.e., the superpotential is *not* the most general one allowed by symmetries.

[7]The "loop" factor for these masses is $\sim \alpha_{GUT}/(4\pi)$, but there is usually an enhancement from group theory and/or large number of messengers which effectively makes the loop factor $\sim \alpha_{GUT}/\pi \sim 10^{-2}$ (for $\alpha_{GUT} \approx 0.04$) – this estimate suffices for comparison to SUGRA mediated masses (see later). The precise expressions for the gauge mediated masses are given in Eqs. (11) and (12).

[8]With the addition of $W_{2,3}$, the flat direction v is a combination of Σ, H, $\bar{H}$, X and $\bar{X}$.

The matter messengers give a positive contribution to the scalar (mass)^2 as usual, but the gauge messenger contribution is typically negative (for all scalars) so that most scalars have negative (mass)^2 at the GUT scale. Of course, in RG scaling to the weak scale, the sfermion (mass)^2 get a positive contribution from the gaugino masses. The gauge mediated MSSM sfermion (other than stop) (mass)^2 at the scale μ are given by (again using the technique of [7]):

$$
\begin{aligned}
m_i^2(\mu) \quad &\approx \quad \frac{1}{16\pi^2}\left(\frac{F_v}{v}\right)^2 \times \\
&\left(\sum_A \frac{2C_A^i}{b_A}\left(\alpha_A^2(\mu)\,(b_6 - b_A)^2 - b_6^2\alpha_6^2\right) + 2C_6^i b_6 \alpha_6^2\right),
\end{aligned}
\tag{12}
$$

where C_A^i is the quadratic Casimir invariant for the scalar i under the gauge group A, i.e., $4/3$, $3/4$ for fundamentals of $SU(3)_c$, $SU(2)_L$ respectively and $3/5\,Y^2$ for $U(1)_Y$. $C_6^i = 35/12$ for fields in $\bar{\mathbf{5}}$ of $SU(5)$ ($\bar{\mathbf{6}}$ of $SU(6)_{GUT}$) and $14/3$ for fields in $\mathbf{10}$ of $SU(5)$ ($\mathbf{15}$ of $SU(6)_{GUT}$). The beta function for $SU(N_c)$ group is defined as $3N_c - N_{f,eff}$, where N_{eff} is the "effective" number of flavors. α_6 is the $SU(6)$ coupling at the GUT scale. With the particle content in Table 1, we get $b_6 = -11$.

In this model, it turns out that the gauge mediated contribution to RH slepton (mass)^2, Eq. (12), is negative at the weak scale, i.e., the positive bino mass contribution (which is *not* an independent parameter) is not enough to make the RH slepton (mass)^2 positive while all other sfermion (mass)^2 are positive due to the larger wino/gluino mass contribution.

So far, the SUGRA contribution to MSSM scalar and gaugino masses has been neglected. The SUGRA contribution to MSSM sfermion (mass)^2 from the operators:

$$
\mathcal{L} \sim \int d^4\theta\, c_1\, \frac{X^\dagger X \Phi_i^\dagger \Phi_j}{M_{Pl}^2},
\tag{13}
$$

where X is any SUSY breaking field (Σ, H_i, X_i etc.) and Φ is a MSSM matter field, is $\sim [F_v/M_{Pl}]^2 \sim 10^{-4}\,[F_v/M_{GUT}]^2$ (assuming $c_1 \sim O(1)$ and $M_{Pl} \sim 2 \times 10^{18}$ GeV). Thus, the SUGRA and gauge mediated contributions (to sfermion masses) are comparable so that the combined RH slepton (mass)^2 can still be positive (provided the SUGRA contribution is positive and a bit larger than the GM contribution) and a realistic spectrum can be achieved. However, the SUGRA contribution (Eq. (13)) violates flavor – in general, the off-diagonal terms in the sfermion (mass)^2 matrices (in flavor space) will be $O([F_v/M_{Pl}]^2)$ which clearly result in too large SUSY contributions to FCNC's. There is also a SUGRA contribution to MSSM gaugino masses ($\sim F_v/M_{Pl}$) comparable to the GM contribution (see (4D equivalent of) Eq. (14)) which, in turn, contributes to scalar masses in RG scaling to the weak scale; this contribution to sfermion (mass)^2 is positive and flavor-conserving.

From the above discussion, it is clear that generic models (i.e., not just the one in [1]) in which the same field breaks SUSY and a GUT symmetry will have the same problem – at the

scale M_{GUT}, there will be a contribution (which is typically negative) to the scalar (mass)^2 from the heavy gauge multiplet so that, at least for the RH slepton, the gauge mediated (mass)^2 at the weak scale might be negative. Also, even if the gauge mediation contribution is positive, the SUGRA contribution (Eq. (13)) to the sfermion (mass)^2 is comparable which leads to flavor violation, in general. It is clear that the latter is a problem also in models of GM with messenger scale close to the GUT scale (having nothing to do with GUT symmetry breaking), for example the model of [8].

3   Improved model using an extra dimension

We now show how to improve the above model, i.e., how to obtain positive and (at the same time) flavor-conserving sfermion (mass)^2 using the framework of gaugino mediated SUSY breaking ($\tilde{g}$MSB) [4, 5].

Consider the following embedding of this model in a 5-dimensional ($5D$) theory. The MSSM matter fields (i.e., N and $\bar{P}_{1,2}$) are localized on a "3-brane" ("MSSM matter" brane) whereas the $SU(6)_{GUT}$ gauge fields and Y, $\bar{Y}$, h, $\bar{h}$ fields propagate in the extra dimension. SUSY breaking fields, i.e., Σ, H, $\bar{H}$, X and $\bar{X}$ and Q, $\bar{Q}$ are localized on a different 3-brane ("SUSY breaking" brane) which is separated from the MSSM matter brane by a distance $R \sim 3\ M^{-1}$ in the extra dimension, where M is the "fundamental" ($5D$) Planck scale. For simplicity we also assume that R is the size of the extra dimension. The $S_{1,2,3}$ fields and the $SU(6)_S$ gauge fields can either propagate in the bulk or be localized on the SUSY breaking brane.

In the effective $5D$ field theory below M ($\sim 10^{18}$ GeV, see later), direct couplings between fields on the matter brane and SUSY breaking brane are forbidden since such couplings are not local [9]. Of course, such operators might be generated by integrating out bulk states with mass $\sim M$, but such effects will suppressed by a Yukawa factor $\sim e^{-RM}$ due to the (position space) propagator of the bulk state [9]. In particular, the coefficient of the operator in Eq. (13) is $\lesssim 10^{-2}$. Thus, the SUGRA contributions to the MSSM squark and slepton (mass)^2 from contact Kähler terms are suppressed by this factor relative to other contributions (see later).

The superpotential couplings written in section 2.1 are all allowed, except for the $N\bar{P}_{1,2}\bar{H}_{1,2}$ coupling in Eq. (10) (N, $\bar{P}_{1,2}$ and $\bar{H}$ fields are localized on different branes). This coupling in the model of section 2.1 gives an $O(M_{GUT})$ mass to $\mathbf{5}$ (under $SU(5)$) of N with a $\bar{\mathbf{5}}$ (under $SU(5)$) in $\bar{P}_{1,2}$ while the massless $\bar{\mathbf{5}}$ in $\bar{P}_{1,2}$ and $\mathbf{10}$ in N are the usual quarks and lepton fields. Thus, in the above framework, a $(\mathbf{5} + \bar{\mathbf{5}})$ (for each generation) is massless in addition to the usual quarks and leptons. The beta-functions of SM gauge group (b_A's) and hence the GM contribution to scalar and gaugino masses (in the $4D$ theory; see later) depends on whether these addtional fields are light or not. We assume that the extra $(\mathbf{5} + \bar{\mathbf{5}})$'s acquire mass $\sim O(M_{GUT})$ by some mechanism.

We require the (usual Higgs doublets in) h, $\bar{h}$ fields to couple to matter fields so that Yukawa couplings can be generated. These Higgs fields should also couple to the GUT symmetry breaking fields (and hence SUSY breaking fields, in this case), which are localized on a different brane, so that the usual Higgs doublet-triplet splitting can be achieved. Thus, Higgs fields h, $\bar{h}$ have to propagate in the bulk.

The MSSM gauginos get a mass from the following ($5D$) SUGRA interactions:

$$\mathcal{L} \sim \int d^2\theta \left(c_2 \frac{\Sigma W_\alpha W^\alpha}{M^2} + c_3 \frac{(X + \bar{X})W_\alpha W^\alpha}{M^2} \right) + h.c., \tag{14}$$

where W_α is a $5D$ gauge field and $c_{2,3} \sim O(1)$. The contribution to MSSM gaugino masses from the singlet is universal while that from Σ is non-universal: $M_1 : M_2 : M_3 = 1 : 5 : -5$. When these operators and also the operators $\sim \int d^4\theta \, X^\dagger X/M^5 \, W_\alpha D^2 W^\alpha$ (which generate a SUSY breaking gaugino wavefunction) are inserted in one-loop diagrams, we get contributions to sfermion (mass)$^2 \sim g_{5D}^2/(4\pi^2) \left[F_v/M^2 \right]^2 1/R^3$ and $\sim g_{(5D)}^2/(4\pi^2) \, F_v^2/M^5 \, 1/R^4$, respectively [4, 5] ($g_{(5D)}$ is the $5D$ gauge coupling) which are *finite* due to the spatial separation of the SUSY breaking and the MSSM matter branes in the $5D$ theory. [10] Using Eqs. (15) and (16) (see below), these one-loop contributions to sfermion (mass)^2 are of order $\alpha_{(4D)}/\pi \left[F_v/M_{Pl} \right]^2 1/(MR)$ and $\alpha_{(4D)}/\pi \left[F_v/M_{Pl} \right]^2 1/(MR)^2$, respectively and thus are negligible compared to GM contribution at M_{GUT} and gaugino mass contribution in RG scaling to the weak scale (see below).

When the extra dimension is compactified (i.e., the Kaluza-Klein (KK) excitations of supergravity and other bulk states are integrated out), we get an effective $4D$ field theory below the scale $R^{-1} \sim 1/3M$. The $4D$ and $5D$ Planck scales are related by

$$M_{Pl}^2 \sim M^3 R \sim 3 \, M^2 \tag{15}$$

so that $M \sim M_{Pl}/\sqrt{3} \sim 10^{18}$ GeV and $R^{-1} \sim M_{Pl}/5 \sim 4 \times 10^{17}$ GeV. [11] *In what follows, we assume $M \sim M_{Pl}$.* It was shown in [10] that the exchange of supergravity Kaluza-Klein (KK)

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[9]For example, the extra $(\mathbf{5} + \bar{\mathbf{5}})$'s might couple to additional fields or another possibility is that the operator $\int d^2\theta \left(N\bar{P}\bar{H}X/M \right)$ is generated with a coupling $\sim e^{-RM}$ by exchange of bulk ("string") states – this operator will gives a mass $O(e^{-3} M_{GUT}/M) \sim 10^{12}$ GeV to the extra $(\mathbf{5} + \bar{\mathbf{5}})$. In the latter case, there is an additonal messenger scale for GM $\sim 10^{12}$ GeV since the $(\mathbf{5} + \bar{\mathbf{5}})$ in N, $\bar{P}_{1,2}$ have a non-supersymmetric spectrum due to the coupling to $\bar{H}$, X – in the $4D$ model of section 2.1 these messengers were at the scale M_{GUT}.

[10]In other words, these contributions to sfermion (mass)^2 are the effect of integrating out the extra dimension, i.e., the Kaluza-Klein excitations of the gauge fields.

[11]For simplicity, we assume that any other extra dimensions have size M^{-1} so that M is the *fundamental* quantum gravity ("string") scale and the inverted hierarchy mechanism generates the hierarchy between M and M_{GUT} (à la the hierarchy between M_{Pl} and M_{GUT} in $4D$). It is easy to extend this framework to one with *more than one* extra dimensions of size *slightly larger* than the fundamental length scale. However, in that case, the fundamental (say, $(4 + n)D$) Planck scale, M, might be smaller than 10^{18} GeV (since $M_{Pl}^2 \sim M^{n+2}R^n$ for n extra dimensions of size R) so that the motivation for the inverted hierarchy mechanism (to explain M_{GUT}/M) is a bit weaker.

excitations also does not lead to contact Kähler terms of order $1/M_{Pl}^2$ (of the form Eq. (13)). The $4D$ and $5D$ gauge couplings are related by

$$g_{(4D)}^2 \sim \frac{g_{(5D)}^2}{R}. \tag{16}$$

The $4D$ (zero-mode) gaugino mass is given by $\sim c_{2,3} \; F_v/M^2 \; 1/R \sim c_{2,3} \; F_v/(\sqrt{3} \; M_{Pl})$, i.e., $O(F_v/M_{Pl})$ [4].

The anomaly mediated contribution to MSSM scalar and gaugino masses [9, 11] is $\sim F_v/M_{Pl} \; \alpha/\pi$ and thus can be neglected in comparison to SUGRA contribution to gaugino masses, GM contributions (at M_{GUT}) to gaugino and scalar masses and gaugino mass contribution to scalar masses in RG scaling to the weak scale (see below). There is also a contribution to scalar (mass)^2 at one-loop $\sim 1/\left(16\pi^2\right) F_v^2/(R^2 \; M_{Pl}^4)$ from integrating out SUGRA KK modes [9]; for $MR \sim 3$, this is comparable to the anomaly mediated contribution and thus can be neglected. [12]

At the scale $M_{GUT} \sim 10^{16}$ GeV [13], the heavy gauge multiplet and the matter messengers are integrated out (as in section 2.2) generating the contributions to the scalar (mass)^2 (at two-loops) and gaugino masses (at one-loop), Eqs. (11) and (12). [14] There are also contributions to sfermion and gaugino masses obtained by replacing the zero-mode ($4D$) gauge fields in the above loop diagrams by KK gauge fields which have masses $\sim kR^{-1}$. The one-loop diagrams with KK gauge fields and gauge messengers give (zero-mode) gaugino masses $\sim \sum_k \; \alpha/\pi \; F_v v/\left(kR^{-1}\right)^2$ (since the R-symmetry breaking scale is v) and the two-loop diagrams with KK gauge fields (and either matter or gauge messengers) give sfermion (mass)$^2 \sim \sum_k \; (\alpha/\pi)^2 \; F_v^2/\left(kR^{-1}\right)^2$. [15] Since $\sum_k \; 1/k^2$ is convergent for the case of one extra dimension, we see that these contributions are suppressed by a factor $\sim 1/\left(M_{GUT}R\right)^2 \sim O(100)$ compared to Eqs. (11) and (12). As mentioned earlier, GM contributions to gaugino and scalar masses (at M_{GUT}) (Eqs. (11) and (12)) and the SUGRA contribution to gaugino mass are all of order F_v/M_{Pl}. In RG scaling to the weak scale, the gaugino masses give an additional (positive) contribution to the sfermion (mass)$^2 \sim \alpha/\pi \; \ln\left(M_{GUT}/m_Z\right) \; [F_v/M_{Pl}]^2 \sim [F_v/M_{Pl}]^2$.

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[12] Both these contributions to sfermion masses and also the one-loop gaugino contribution in the $5D$ theory mentioned above are flavor-conserving.

[13] As before, the GUT scale is determined by the one-loop corrections to the wavefunction (Z) of Σ. Between the energy scales M and R^{-1}, the $SU(6)_{GUT}$ gauge coupling (rather α_6^{-1}) (and similarly Z, $\lambda_{Q,\Sigma}$) "runs" with a power of energy since the gauge theory is $5D$ whereas below $R^{-1} \sim 4 \times 10^{17}$ GeV, we have the usual ($4D$) RG scaling. In any case, the inverted hierarchy mechanism can result in a minimum of the potential at $v \sim 10^{16}$ GeV $\ll M$; most of the RG scaling of Z, g_6 and $\lambda_{\Sigma,Q}$ from M to $\sim 10^{16}$ GeV is the usual $4D$ evolution.

[14] In computing these "threshold" corrections, the SUGRA contribution $\sim F_v/M_{Pl}$ to the SUSY breaking masses of the heavy gauge multiplet and the matter messengers (Q, $\bar{Q}$ etc.) can be neglected in comparison to the contribution $\sim F_v/M_{GUT}$ from direct coupling to v.

[15] Strictly speaking, since this is the effect of integrating out the KK gauge fields, these contributions to sfermion and gaugino masses appear at/above the scale $\sim R^{-1}$.

It is clear that if $R \sim 3M^{-1}$, then the SUGRA contribution to the sfermion (mass)^2 at the high scale from contact Kähler terms (Eq. (13) with a coefficient $\sim e^{-MR}$) can be neglected in comparison to the GM contribution at M_{GUT} and the contribution generated by gaugino masses in RG scaling. This means that sfermion masses conserve flavor, i.e., sfermion with the same gauge quantum numbers are degenerate at the $O(e^{-3}) \sim$ percent level. [16]

As mentioned earlier, if the SUGRA contribution to sfermion *and* gaugino masses is neglected, then the (gauge mediated contribution to) RH slepton (mass)^2 at the weak scale is negative. However, in this ($5D$) framework, while the SUGRA contribution to RH slepton mass at the high scale (Eq. (13)) is suppressed, we have to include the SUGRA contribution ($\sim F_v/M_{Pl}$) to the bino mass which, in turn, generates in RG scaling to the weak scale an *additional* (compared to the pure GM case) positive contribution $\sim [F_v/M_{Pl}]^2$ to the RH slepton (mass)^2. This can result in a positive RH slepton (mass)^2 at the weak scale, i.e., unlike the pure GM case, due to the SUGRA contribution to the bino mass, the bino mass contribution is (effectively) *independent* of (and of the same order as) the GM contribution to the RH slepton mass. [17]

It is obvious that the same framework (extra dimension of size $R \sim 3M^{-1}$) can be used to suppress (flavor-violating) SUGRA contribution to sfermion masses (Eq. (13)) in *any* model where the GM and SUGRA contributions are comparable, for example, a model of GM with messenger scale close to the GUT scale (where the GM contribution to sfermion (mass)^2 is, say, *positive*) [8]. As mentioned earlier, *any* model with GUT and SUSY breaking by the same field is likely to have negative GM contribution to sfermion (say, RH slepton) (mass)^2; it is clear that (in addition to suppressing the SUGRA contribution to sfermion masses) the above idea (the SUGRA contribution to bino mass) can be used to obtain positive RH slepton (mass)^2.

A slightly modified version of the above model is obtained by gauging only the $SU(5)$ subgroup of the $SU(6)_{GUT}$ symmetry [1]. The superpotential is given by $W_1 + W'$ with

$$W' = h \left(\lambda_1 \Sigma_1 + \lambda_{24} \Sigma_{24} \right) \Sigma_{\bar{5}} + \bar{h} \left(\bar{\lambda}_1 \Sigma_1 + \bar{\lambda}_{24} \Sigma_{24} \right) \Sigma_5, \tag{17}$$

where Σ_r's refer to components of Σ transforming as **r** under $SU(5)_{local}$ and h, $\bar{h}$ form a (**5**+**$\bar{5}$**) of $SU(5)_{local}$. Σ and Q, $\bar{Q}$ are as usual localized on the SUSY breaking brane with h, $\bar{h}$ fields in the bulk. In the absence of W', Σ has a pair of massless color triplets which are Nambu-Goldstone fields since the full $SU(6)_{GUT}$ is not gauged. W' gives masses to these triplets with those in h, $\bar{h}$. Since $\langle \Sigma_1 \rangle \sim$ diag $[1, 1, 1, 1, 1]$ whereas $\langle \Sigma_{24} \rangle \sim$ diag $[3, 3, -2, -2, -2]$ in $SU(5)$ space, we

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[16]This degeneracy is sufficient to evade limits from *CP-conserving* flavor-violating processes, for example, $\mu \to e\gamma$, Δm_K etc. But, if the SUGRA mediated sfermion (mass)^2 in Eq. (13) have $O(1)$ phases, then we need $R \sim 6\ M^{-1}$ to obtain degeneracy at the $\sim 0.1\ \%$ level so that SUSY contributions to *CP and* flavor-violating processes are suppressed.

[17]Thus, the role of the extra dimension here (as in [4, 5]) is to suppress the (flavor-violating) SUGRA contribution to sfermion masses while allowing SUGRA contribution to gaugino masses (in particular, in this case, bino mass). It is clear that any other framework which provides these boundary conditions also suffices.

can (fine-)tune the $\lambda_{1,24}$ couplings so that the weak doublets in h, $\bar{h}$ are massless; these will be the usual Higgs doublets.

In this version of the model, doublet-triplet splitting (although "technically natural") is fine-tuned. Also, there is a global $SU(6)$ symmetry on the SUSY breaking brane, i.e., W_1 is $SU(6)$ symmetric, but the couplings of the bulk fields, h, $\bar{h}$ to Σ (Eq. (17)) break this to $SU(5)_{local}$. The nice feature compared to the earlier model is that quarks and leptons are contained in the usual ($\bar{\mathbf{5}} + \mathbf{10}$) of $SU(5)$ (localized on the matter brane) and so, unlike the gauged $SU(6)$ model, "splitting" of matter superfields is not required. As mentioned earlier, in the gauged $SU(6)$ model, quarks and leptons are contained in ($\mathbf{15} + \bar{\mathbf{6}} + \bar{\mathbf{6}}$) of $SU(6)$ and a ($\bar{\mathbf{5}} + \mathbf{5}$) (under $SU(5)$) are made heavy by coupling to $\bar{H}$ in the $4D$ model – this coupling is not allowed (at the renormalizable level) in the $5D$ model since $\bar{H}$ and matter fields are localized on different branes. In the $SU(5)$ model, there are SUGRA contributions to MSSM gaugino masses from both singlet (Σ_1) and adjoint (Σ_{24}) SUSY breaking fields (in addition to the GM contribution). Thus, as in the gauged $SU(6)$ model, the MSSM gaugino masses (M_1, M_2 and M_3) are free parameters (see section 4).

Comments on other models: Some of the models of gauge mediation in the literature ([12, 13]) also have gauge messengers and hence negative MSSM scalar (mass)^2 at the messenger scale, M_{mess} (the SUGRA contribution is much smaller than the GM contribution if $M_{mess} \lesssim 10^{15}$ GeV). As usual, the gaugino masses give a positive contribution in RG scaling to the weak scale; however, at least RH sleptons still have negative (mass)^2 [7]. The framework of $\tilde{g}$MSB can also be used to "resurrect" these models as follows. [18] Suppose the SUSY breaking fields are localized on a brane different than the MSSM matter brane in an extra dimension with a compactification scale (R^{-1}) slightly smaller than M_{mess} (with gauge fields in the bulk). [19] Then the two-loop (negative) gauge mediated contribution to sfermion (mass)^2 at M_{mess} is *suppressed* (in the $5D$ theory) by a factor $1/(RM_{mess})^2 \sim 1/25$ (if $R \sim 5\ M_{mess}^{-1}$) relative to the $4D$ result, i.e., it is $\sim 1/25\ [\alpha/\pi]^2\ [F/M_{mess}]^2$ [14, 4]. The gaugino masses are the same as in $4D$ [4] ($\sim \alpha/\pi\ F/M_{mess}$) and, in turn, give a positive $O(\alpha/\pi\ F/M_{mess})^2$ contribution to sfermion (mass)^2 in RG scaling to low scales (provided $M_{mess} \gg m_Z$, i.e, RG logarithm is large enough). Since the (negative) sfermion (mass)^2 at M_{mess} is small compared to this RG contribution, the sfermion (including RH slepton) (mass)^2 at the weak scale can be positive.

Above the scale M_{mess}, the theory is $5D$ with MSSM matter fields on a brane and gauge fields in bulk. If Higgs fields are also in the bulk, then in this framework gauge coupling unification is (approximately) preserved (with lower unification scale) [15].

We can use a similar idea to suppress (negative) gauge mediated sfermion (mass)^2 (at M_{GUT})

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[18] This was partly hinted in [4].

[19] As before, we assume that the two branes are maximally separated in the extra dimension.

in the GUT model, i.e, we can invoke an extra dimension slightly larger than (inverse of) the GUT scale (which is M_{mess} in this case). This will suppress *both* the SUGRA and (negative) GM contributions to sfermion (mass)^2 at the high scale. But, with (say) $R \sim 5\, M_{GUT}^{-1} \sim (4 \times 10^{15}\text{ GeV})^{-1}$, we get the $5D$ Planck scale $M \sim 10\, M_{GUT}$ (using $M_{Pl}^2 \sim M^3 R$) so that the motivation for the inverted hierarchy mechanism (to generate M_{GUT} smaller than the fundamental Planck scale by a factor of ~ 100 as before) is a bit weaker. [20] In addition a new "hierarchy", $R \sim 50\, M^{-1}$, would have to be explained. [21] In any case, in the model with $R \sim 5\, M_{GUT}^{-1}$, the SUGRA and GM contributions to gaugino masses will be (roughly) same as in the model with $R \sim 3M^{-1}$, i.e., $M_{1,2,3}$ are free parameters which will generate positive sfermion (mass)^2 in RG scaling to the weak scale – the only difference is that in the model with $R \sim 3M^{-1}$, there is a (negative) GM contribution to scalar (mass)^2 at M_{GUT}. The phenomeonlogy of these two models should be similar (see section 4).

It is also clear from the discussion in section 2.2 that, in general, in models with (dominant) SUSY breaking in a GUT non-singlet (denoted by Σ), there is a contribution to MSSM gaugino masses at one-loop (and to scalar (mass)^2 at two-loop) from the coupling to gauge messengers – to repeat, these are the heavy gauge multiplets (with mass $\sim M_{GUT}$) which have a non-supersymmetric spectrum since the SUSY breaking field (Σ) transforms under the GUT gauge group. The (contributions to) MSSM gaugino masses generated at one-loop by integrating out gauge messengers (at the scale M_{GUT}) are generically (i.e., barring accidental cancellations) given by $\sim \alpha/\pi\, F_\Sigma\, M_{\not R}/M_{GUT}^2$, where $M_{\not R}$ is the R-symmetry breaking scale – thus the size of this contribution is model-*dependent* due to $M_{\not R}$. The point is that if the field Σ also breaks the GUT symmetry (down to the SM gauge group), i.e., if the vev of the scalar component

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[20]An even larger extra dimension (in which *only* gravity propagates) can be used to lower M all the way to M_{GUT} [16] so that there is *no* hierarchy between the fundamental Planck scale and GUT scale – of course, in this case (as in the case with $R \sim 5\, M_{GUT}^{-1}$) one has to explain the "hiererchy" between R^{-1} and M. Here (as in [1]), instead we would like to "explain" $M_{GUT} \sim 10^{-2} M$ using the inverted hierarchy mechanism.

[21]In this model, the RG scaling of SM gauge couplings above the energy scale $R^{-1} \sim 4 \times 10^{15}$ GeV is $5D$ – unification of SM gauge couplings still occurs (Higgs fields also propagate in $5D$) but at a scale lower than the usual M_{GUT} by a factor of ~ 2 [15]. For illustrative purposes, we assumed above that the unification scale is still $M_{GUT} \sim 2 \times 10^{16}$ GeV. In general, we can choose the compactification scale, R^{-1}, to be much smaller than 10^{15} GeV so that unification occurs at a scale, M'_{GUT}, *which depends on* R^{-1} and which is much smaller than the usual $M_{GUT} \sim 10^{16}$ GeV [15]. However, in this case, one has to "explain" why the compactification scale, R^{-1}, is correlated with the GUT scale (M'_{GUT}) which (in the context of the model in this paper), in turn, is determined by a modulus field (i.e, which is *not* a fundamental scale). Also, if $R \gg M^{-1}$, then the $5D$ gauge couplings (assuming that $4D$ gauge couplings are $O(1)$) might become non-perturbative (larger than their strong coupling value) (see Eq. (16)). Of course, one faces a similar issue(s) in trying to "save" the GM models above except that in that case the correlation between M_{mess} (which is presumably fixed by a modulus field also) and R^{-1} is weaker since we only require $R \gtrsim 5\, M_{mess}^{-1}$. In contrast, in the GUT model with $R \sim 3M^{-1}$ there is only a "modest" hierarchy between R^{-1} and the $5D$ Planck scale M (which, as mentioned earlier, is assumed to be *fundamental*).

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of Σ (v_Σ) is $O(M_{GUT})$ (as in our model), then $M_{\tilde{R}} \sim M_{GUT}$. Therefore, in a model with $F_\Sigma \neq 0$, *if* v_Σ (or, in general, $M_{\tilde{R}}$) is $O(M_{GUT})$, then this gauge messenger contribution to MSSM gaugino masses is *comparable* to (and *independent* of) the SUGRA contribution (from the operator $\int d^2\theta \, \Sigma/M_{Pl} \, W_\alpha W^\alpha + h.c.$) [22] $\sim F_\Sigma/M_{Pl}$ (and is also non-universal, in general). [23] Thus, as in our $SU(6)$ GUT model, the MSSM gaugino mass relations are modified from that expected with only SUGRA contribution – in fact the model becomes *less* predictive (as far as gaugino masses are concerned) since there is an extra parameter corresponding to the gauge messenger contribution.

There have been some recent studies of MSSM gaugino masses in a scenario with SUSY breaking by a GUT non-singlet so that SUGRA contribution to gaugino masses is non-universal [17] – in these studies, the above gauge messenger contribution has *not* been mentioned. In the first reference in [17], it is *assumed* that $v_\Sigma \sim 0$, i.e., the SUSY breaking field has a small vev in its scalar component. In this case, it is possible that $M_{\tilde{R}} \ll M_{GUT}$ so that the gauge messenger contribution to MSSM gaugino masses is small compared to the SUGRA contribution. Nonetheless, (in general) in order to be sure that the gauge messenger contribution to MSSM gaugino masses is smaller than the SUGRA contribution, the complete model has to be analysed to check that v_Σ (or $M_{\tilde{R}}$) $\ll M_{GUT}$. Also, a contribution to MSSM scalar (mass)^2 $\sim [\alpha/\pi]^2 \, [F_\Sigma/M_{GUT}]^2$ is generated at two-loops by integrating out gauge messengers – there is *no* suppression due to $M_{\tilde{R}}$ unlike in the case of MSSM gaugino masses (since scalar (mass)^2 do not break R-symmetry). Thus, the gauge messenger contribution to scalar (mass)^2 is comparable to (and again, independent of) the SUGRA contribution to scalar (mass)^2 $\sim [F_\Sigma/M_{Pl}]^2$ (due to contact Kähler terms) *even if* $v_\Sigma \sim 0$ (i.e., the size of this contribution is model-*in*dependent). Furthermore, the gauge messenger contribution depends on gauge quantum numbers and thus it is different for squarks and sleptons (as in our $SU(6)$ GUT model), although it is flavor-conserving.

4   Sparticle spectrum

We now present briefly a sample sparticle spectrum in the $5D$ model.

4.1   Parameters of the model

In this model, the MSSM sfermion masses (at M_{GUT}) and gaugino masses are determined by three parameters – F_v/v ($v \sim M_{GUT}$), c_2 and c_3. The GM contribution (Eqs. (11) and (12))

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[22]We assume that Σ is in a representation which appears in the symmetric product of two adjoints.

[23]There might also be other GM contribution to gaugino (and scalar) masses from, say, "matter" messengers (as in our model).

can be written in terms of F_v/v (assuming that the beta-functions [24] and gauge couplings are fixed) and the SUGRA contrbution to MSSM gaugino masses, Eq. (14), depends on F_v/v and $c_{2,3}$ (any uncertainty in the ratio of $v \sim M_{GUT}$ and M (or M_{Pl}) can be absorbed into $c_{2,3}$). For convenience, we choose the gaugino masses at the GUT scale, M_1, M_2 and M_3 (which are combinations of these parameters) to be the free parameters. [25] Using Eqs. (11) and (14), we get:

$$\frac{1}{36}\left(-5M_1 + 3M_2 + 2M_3\right) = \frac{\alpha_6}{4\pi}\frac{F_v}{v} \tag{18}$$

which parametrizes the GM contribution. This relation is used to determine the GM contribution to scalar (mass)^2 at the GUT scale in terms of M_A's using Eq. (12) with $\mu \approx M_{GUT}$. The scalar masses are then evolved to the weak scale using the one-loop RG equations.

We note that the SUSY-GUT prediction for $\sin^2\theta_w$ (in terms of α_s and α_{em}) is affected by the operator $W_\alpha W^\alpha \Sigma/M^2$ in Eq. (14) [18] – in this model, this effect (which is at the M_{GUT}/M_{Pl}, i.e., percent level) is related to SUGRA contribution to gaugino mass. The method used to give mass to extra doublets in H, $\bar{H}$ (see section 2.1) also affects the prediction for $\sin^2\theta_w$ – in this case this pair of doublets gets a mass of $O(M_{GUT}^2/M) < M_{GUT}$ (from the superpotential W_4) which shifts the $\sin^2\theta_w$ prediction by about a percent. Since we wish to illustrate the main features of the spectrum in this paper, we will neglect these effects (which might cancel each other).

We also neglect RG scaling of sfermion masses (at one-loop due to SUGRA contribution to gaugino masses and at two-loops due to SUGRA contribution ($\sim F_v/M_{Pl}$) to Q, $\bar{Q}$ etc. masses) between the (5D) Planck scale M and the GUT scale (sfermion masses are negligible at the scale M) [26] – the RG logarithm $\sim \ln\left(M/M_{GUT}\right)$ is much smaller than that for RG scaling between GUT and weak scale (of course, the larger group theory factors above M_{GUT} might compensate for the smaller RG logarithm as emphasized in the context of extra dimensional models in [20]). [27] The (SUGRA mediated) gaugino masses also run between M and M_{GUT}; however since M_A/α_A is RG-invariant (at one-loop), the ratio of the MSSM gaugino masses remains the same in this RG scaling since the gauge couplings are unified (of course, at M_{GUT}, the MSSM gauginos get additional contribution to their masses (Eq. (11)).

Since the usual Higgs doublets propagate in the extra dimension, soft SUSY breaking Higgs

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[24]As mentioned earlier, even though the $N\bar{P}\bar{H}$ coupling is not allowed (at the renormalizable level) in the 5D model, we assume that the additional $(\mathbf{5} + \bar{\mathbf{5}})$ in N, $\bar{P}$'s have a mass of M_{GUT} and so the values of the beta-functions are the same as in section 2.1 (the 4D model), i.e., $b_6 = -11$ and b_A's are the usual MSSM beta-functions.

[25]Note that if only the adjoint field (and not singlets) breaks SUSY, then $c_3 = 0$ in Eq. (14) and one has only two free parameters.

[26]This contribution is positive and flavor-conserving – thus it makes sfermions with same gauge quantum numbers more degenerate and in particular the RH slepton heavier (see later).

[27]A detailed study of the phenomenology in this GUT model (including the effects of RG scaling between M and M_{GUT}) is in progress.

(mass)^2 and also $B\mu$ and μ (Giudice-Masiero mechanism [19]) are generated by the SUGRA interactions:

$$\mathcal{L} \;\sim\; \int d^4\theta \left(\frac{1}{M^3} h^\dagger h \left[\Sigma^\dagger \Sigma + .. \right] + \frac{1}{M^3} \bar{h} \bar{h}^\dagger \left[X^\dagger X + .. \right] \right) +$$
$$\int d^4\theta \left(h\bar{h} \frac{1}{M^2} \left[\Sigma^\dagger + X^\dagger + \bar{X}^\dagger \right] + h\bar{h} \frac{1}{M^3} \left[\Sigma^\dagger \Sigma + .. \right] + h.c. \right), \tag{19}$$

where h, $\bar{h}$ are $5D$ fields. The couplings of zero-modes of h, $\bar{h}$ (which correspond to the light $4D$ Higgs fields) are suppressed by a factor of $\sqrt{R}$ (to account for the normalization of the zero mode) so that $m^2_{H_{u,d},SUGRA} \sim F_v^2/M^3 \; 1/R$, $\mu \sim F_v/M^2 \; 1/R$ and $B\mu \sim F_v^2/M^3 \; 1/R$. Since $MR \sim 3$ and $M_{Pl}^2 \sim M^3 R$, we see that $m^2_{H_{u,d},SUGRA}$, μ^2, $B\mu$ are all of order $(F_v/M_{Pl})^2$, i.e, of the same order as, but independent of, gaugino and sfermion masses (they are also independent of each other). Of course, the Higgs doublets also get soft (mass)^2 from GM (Eq. (12), which are related to the other sfermion and gaugino masses. We choose the Higgs soft (mass)^2 at the GUT scale (which are the sum of the SUGRA and GM contributions) to be free parameters, denoted by $m^2_{H_u}$ and $m^2_{H_d}$.

In this model, the gauginos of the heavy gauge multiplet have a SUSY breaking mass $\sim F_v/M_{GUT}$ since the SUSY breaking field is an adjoint under the GUT gauge group. This generates trilinear (MSSM) scalar terms of $O(\alpha/\pi F_v/M_{GUT}) \sim F_v/M_{Pl}$ at the GUT scale (when the heavy gauginos are integrated out). The exact expression is [7]:

$$V \;\ni\; \sum_i A_i Q_i \partial_{Q_i} W(Q), \tag{20}$$
$$A_i(\mu_{RG}) \;=\; \frac{\partial \ln Z_{Q_i}(v^\dagger, v, \mu_{RG})}{\partial \ln v} \frac{F_v}{v}.$$

In this case, we have

$$A_i(M_{GUT}) \;=\; \frac{F_v}{v} \frac{\alpha_6}{4\pi} \left(2C_6^i - 2\sum_A C_A^i \right). \tag{21}$$

We neglect all Yukawa couplings other than the top quark coupling and so only the coupling $\lambda_t A_t H_u \tilde{Q}_3 \tilde{t}^c$ is non-zero and is given by

$$A_t(M_{GUT}) \;=\; 15.3 \frac{F_v}{v} \frac{\alpha_6}{4\pi}. \tag{22}$$

Thus, the A-term at the GUT scale depends only on F_v/v (and gauge couplings), i.e., it is *not* an independent parameter – in particular, there is no SUGRA contribution to A_t since the top squark and the SUSY breaking fields are localized on separate branes. [28]

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[28]As mentioned earlier, we neglect RG scaling between GUT and Planck scales; this effect does generate (due to non-vanishing gaugino masses) a small A_t term at the GUT scale which depends on the SUGRA contribution to gaugino masses, i.e., F_v/v, c_2 and c_3.

$M_{\chi_1^0}$	105, 120
$M_{\chi_2^0}$	165
$M_{\chi_3^0}$	190
$M_{\chi_4^0}$	290
$M_{\chi_1^+}$	145, 170
$M_{\chi_2^+}$	290
$m_{\tilde{e}_R}$	115
$m_{\tilde{e}_L, \tilde{\nu}_L}$	210, 200
$M_{\tilde{g}}$	365
$m_{\tilde{u}_R}$	315
$m_{\tilde{d}_R}$	320
$m_{\tilde{u}_L, \tilde{d}_L}$	375, 380
$m_{\tilde{t}_{1,2}}$	190, 400

Table 2: Sample spectrum in the model for renormalization scale $\mu_{RG} \approx 500$ GeV and the input parameters: $M_1 = M_2 = -300$, $M_3 = -150$, $m_{H_u}^2 = (150)^2$, $m_{H_d}^2 = (300)^2$ and $\tan\beta = 5$ which give $\mu = 180$ (all masses in GeV). The two values for $M_{\chi_1^0}$ and $M_{\chi_1^+}$ are for different signs of μ (the other neutralino/chargino and stop masses do not depend strongly on the sign of μ).

Thus, the *fundamental* parameters in this model are: F_v/v, $c_{2,3}$, $m_{H_{u,d}}^2$, $B\mu$, μ and λ_t (top quark Yukawa coupling). Two of these parameters are fixed by the observed values of m_Z and m_t so that the *free* (i.e., input) parameters can be chosen to be $M_{1,2,3}$ (which, as explained earlier are combinations of F_v/v and $c_{2,3}$), $m_{H_{u,d}}^2$ and $\tan\beta$; μ and $B\mu$ can then be determined in terms of these parameters as usual using the minimization conditions for the Higgs potential.

4.2   Sample sparticle spectrum

In Table 2, a sample spectrum is presented for the input parameters $M_1 = M_2 = -300$ GeV and $M_3 = -150$ GeV, $m_{H_u}^2 = (150 \text{ GeV})^2$, $m_{H_d}^2 = (300 \text{ GeV})^2$ and $\tan\beta = 5$. We have included the electroweak D-term and Fayet-Illiopoulos (FI) D-term contributions to scalar (mass)^2 which are given by $m_Z^2 \cos 2\beta \left(T_3 - Q \sin^2\theta_w \right)$ and $-0.053\, Y \left(m_{H_u}^2 - m_{H_d}^2 \right)$ [29], respectively. [30] The mixing between the top squarks and the one-loop corrections to the effective Higgs potential (due to top quark and top squark masses only) have been included.

Some of the characteristic features of the spectrum are as follows.

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[29]The FI D-term contribution in $\tilde{g}$MSB was emphasized in [5].

[30]We assume for simplicity that there is no D-term contribution from the breaking of the extra $U(1)$ (of $SU(6)$) at M_{GUT}.

Gaugino masses are non-universal in general since both the GM contribution and a part of the SUGRA contribution (due to the operator $\int d^2\theta \, c_2 \, \Sigma/M^2 \, W_\alpha W^\alpha$) are non-universal (the relative GM contributions to gaugino masses (Eq. (11)) are model-dependent due to dependence on b_6). Models with non-universal gaugino masses (at the GUT/Planck scale) have been studied earlier [21]. In most of these models, sfermion masses are independent parameters whereas in our GUT model the sfermion masses are determined in terms of the gaugino masses ($M_{1,2,3}$). [31]

No-scale SUGRA models have vanishing scalar masses at M_{Pl} and gaugino masses (which can be non-universal) as usual drive sfermion (mass)^2 positive in RG scaling to the weak scale ($\tilde{g}$MSB with non-universal gaugino masses has similar boundary conditions). However, in the GUT model studied here, there is a GM contribution to scalar masses at the GUT scale and also there are SUGRA contributions to Higgs soft masses (which, in turn, result in a FI D-term contribution to sfermion masses at the weak scale). Thus, the GUT model can (in principle) be distinguished from no-scale SUGRA models with non-universal gaugino masses by precision sparticle spectroscopy.

The (GM contribution to) RH slepton (mass)^2 at the GUT scale is negative (Eq. (12)) and therefore RH slepton (its (mass)^2 is driven positive by bino mass) and χ_1^0 (which is roughly the bino) are close in mass. [32] For the same reason, the mass splitiing between the left-handed slepton and the RH slepton is large [33] even though we have chosen $M_1 = M_2$ for the above sample spectrum, i.e., usually we expect $m_{\tilde{e}_L} - m_{\tilde{e}_R}$ to be large only if $M_2 > M_1$ (due to gaugino mass contributions in RG scaling). As mentioned earlier, if we include RG scaling between M and M_{GUT}, then *all* sfermions will be heavier leading to a *larger* mass splitting between RH slepton and χ_1^0 (which will be the lightest supersymmetric particle (LSP)) whereas the large mass splitting between $\tilde{e}_L$ and $\tilde{e}_R$ (which is due to the negative GM contribution to $m_{\tilde{e}_R}^2$ at M_{GUT}) will remain about the *same*.

The lower limit on the RH slepton mass ~ 90 GeV fixes (roughly) a minimum value for M_1. But since the three gaugino masses are independent parameters, M_3 can be smaller than $M_{1,2}$ so that there is not much of a hierarchy (in masses) between squarks/gluino and sleptons as seen in Table 2. Also, since M_3 can be smaller than $M_{1,2}$ and also (GM mediated) stop (and other squark) (mass)^2 are small and negative at the GUT scale, $|m_{H_u}^2|$ at the weak scale (as usual the up-type Higgs (mass)^2 is driven negative by stop (mass)^2 and gluino mass) and hence μ can be small (in this case ~ 180 GeV), thus reducing the fine-tuning in electroweak symmetry breaking

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[31]In "D-brane" models [22], it is possible that $M_2 \neq M_3$ if $SU(3)_c$ and $SU(2)_w$ originate from different D-brane sectors. Since quark doublets transform under all three SM gauge groups, $U(1)_Y$ has to orginate in either of these two sectors – this implies that $M_1 = M_2$ or $M_1 = M_3$, unlike the GUT model where it is possible that *all three* gaugino masses are different.

[32]In this sample spectrum, the FI D-term (positive for RH slepton) makes the RH slepton (slightly) heavier than χ_1^0 (for one sign of μ).

[33]If the GM contribution at M_{GUT} is neglected, then we get $m_{\tilde{e}_R} \approx 140$ GeV while $m_{\tilde{e}_L}$ remains about the same.

– small μ also results in "light" chargino/neutralino. [34]

This should be compared to $\tilde{g}$MSB with *universal* gaugino mass [4, 5] where a minimum value for M_1 fixed by RH slepton mass implies a minimum value for M_3 (typically $\stackrel{>}{\sim}$ 200 GeV) which results in a larger hierarchy betwen slepton and squark/gluino masses and also larger $|m_{H_u}^2|$ and hence fine tuning (due to larger μ). In a minimal gauge mediation model also, there is a large hierarchy between squark/gluino masses and slepton masses (since masses are proportional to gauge couplings). In minimal SUGRA mediated SUSY breaking (with universal scalar mass, m_0, and universal gaugino mass, $M_{1/2}$) it is possible to have small hierarchy between sleptons and squarks/gluino. In any case, non-universal gaugino masses distinguishes these models from our model.

Of course, we expect that with extra parameters as compared to a minimal model (especially non-universal gaugino masses) such a spectrum can be attained. However in this model these extra parameters are *not ad hoc*, but are *well-motivated* – they are justified by the way SUSY is broken in the model.

5   Summary

A model in which the *same* scalar potential breaks SUSY and a GUT symmetry was presented in [1] – this model has *dynamical* origins for *both* SUSY breaking and GUT scales. In this model, the SUGRA and gauge mediated contributions to scalar and gaugino masses are comparable – this enables a viable spectrum to be attained since the gauge mediated contribution to RH slepton (mass)^2 by itself is negative. But, the flip side is that the SUGRA contribution to sfermion masses (from non-renormalizable contact Kähler terms) results in flavor violation.

In this paper, we suggested that this "problem" will be present in *any* model in which the same field breaks SUSY and a GUT symmetry and demonstrated that, using an extra spatial dimension, positive and (at the same time) flavor-conserving sfermion (mass)^2 can be obtained in this model. The model has *non*-universal gaugino masses and sfermion masses are predicted in terms of gaugino masses. The hierarchy between squark/gluino masses and slepton masses can be small and (typically) a large mass splitting between RH and LH slepton is expected.

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[34] As mentioned earlier, RG scaling between M_{GUT} and M will give a positive contribution to scalar (mass)^2 due to (SUGRA mediated) gaugino masses – this contribution is about the same for all scalars (squarks, sleptons and Higgs) because of the unified gauge symmetry, unlike the case of RG scaling *below* the GUT scale where, since $\alpha_3 > \alpha_{1,2}$ the gluino mass contribution (to squark masses) is larger (assuming universal gaugino masses). Thus, the above features, i.e., the "small" hierarchy between squarks and sleptons and small μ will *not* be affected by the inclusion of this effect.

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